

1 **Modular Joint for the Accelerated Fabrication**

2 **and Erection of Steel Bridges**

3 Mirela D. Tumbeva, S.M.ASCE¹; Ashley P. Thrall, A.M.ASCE²; Theodore P. Zoli, P.E., M.ASCE³

4 **ABSTRACT**

5 This paper introduces a new strategy for the accelerated fabrication and erection of steel bridges:
6 a modular joint. The modular joint is a prefabricated, nodal connector comprised of a weld-
7 ment/built up section of webs and flanges that includes a starter segment for each connecting
8 member. It joins standard rolled wide flange sections through bolted splice connections in double
9 shear. Flanges and webs are connected independently, forming a moment-resisting connection.
10 This provides flexural stiffness for truss-like or beam-like behavior and provides the potential for
11 the structure to tolerate member loss. The flange splice plates connecting the joint and any member
12 can be bent to varying angles to achieve a variable depth geometry. This is a “kit-of-parts” ap-
13 proach, where members are standard sections and the prefabricated modular joint can be repeated
14 throughout a single structure and also used for many structures. While this approach retains all
15 the advantages of modular construction (e.g. prefabrication, mass-production, rapid erection, and
16 reusability), it overcomes the prime deficiency of the existing technologies that a fixed panel size
17 limits the span length. This paper investigates this approach through (1) developing a methodology
18 to achieve rational constant- and variable-depth bridge forms, (2) performing structural optimiza-
19 tion for minimum self-weight while meeting structural performance demands and transportability
20 criteria, and (3) demonstrating the promise of this approach through detailed finite element numer-
21 ical analyses.

¹Ph.D. Candidate, Kinetic Structures Laboratory, Department of Civil and Environmental Engineering and Earth Sciences, University of Notre Dame, Notre Dame, IN 46556. E-mail: mtumbeva@nd.edu

²Myron and Rosemary Noble Associate Professor of Structural Engineering, Kinetic Structures Laboratory, Department of Civil and Environmental Engineering and Earth Sciences, University of Notre Dame, Notre Dame, IN 46556. (corresponding author) E-mail: athrall@nd.edu

³National Bridge Chief Engineer, HNTB Corporation, Empire State Building, 350 5th Ave., 57th Floor, New York, NY 10118. E-mail: tzoli@hntb.com

22 **Key Words:** Modular construction, Accelerated bridge construction, Steel truss, Optimization

23 INTRODUCTION

24 Modular structures (i.e., structures comprised of identical repeated components) provide advan-
25 tages as components can be prefabricated and mass-produced, resulting in cost and time savings
26 as well as improved quality. These benefits are compounded when the same module can be used
27 for many structures. Existing approaches for modular bridges (e.g., Bailey, Acrow) are comprised
28 of prefabricated, rectangular steel panels [typically 3.05 m (10 ft) in length] that are connected by
29 pins arranged in a longitudinal configuration to form a girder-type bridge, with additional capacity
30 and/or span length achieved by stacking panels transversely and vertically [achieving spans up to
31 approximately 91.4 m (300 ft)] (Figure 1) (Joiner, 2001; Russell and Thrall, 2013). A primary lim-
32 itation of these existing approaches is that a fixed panel size limits the span length. Specifically, the
33 span is limited by buckling of the upper chord. While lateral bracing can be utilized, it is expensive
34 and can be time-consuming to install (Gerbo et al., 2016a; Wang et al., 2016).

35 In comparison to girder-type bridges, trusses can be more efficient for longer spans [exceeding
36 91.4 m (300 ft)]. However, gusset plates are typically used to join members, resulting in the fol-
37 lowing deficiencies: (1) inefficiency - as bolts are used in single shear, a large number is required
38 leading to increased time and cost of fabrication and erection, as well as reduced net section of the
39 plate, (2) poor durability - as debris can become trapped and connections are subjected to deic-
40 ing salts, (3) difficult inspection, (4) difficult maintenance as connections are laborious to replace
41 or repair, and (5) challenging fabrication (Covington et al., 2013). These deficiencies have been
42 overcome in the “gussetless” Memorial Bridge connecting Portsmouth, NH and Kittery, ME by
43 using a “knuckle” (Figure 2A). The knuckle enables (1) the use of splice connections in double
44 shear, reducing the number of bolts, (2) the splices to be located away from the concentric in-
45 tersection of members to facilitate inspection, maintenance, and repair, and (3) the use of wide
46 flange members for the diagonals. The strong axis orientation of the wide flange members and the
47 moment-resisting connections between the members results in increased reliability and redundancy
48 as chords can carry load in bending if a diagonal is lost (Covington et al., 2013). The knuckles

49 are easily fabricated from flat steel plate, with the flanges being cold bent and welded to the webs
50 (Figure 3). Field installation required only bolted connections. This approach offered significant
51 advantages in fabrication and erection time, as well as cost (Covington et al., 2013). The bridge
52 contract was awarded based on a Best Value Award Determination, which included scheduled
53 completion over 4 months faster than competitors, a total project value of \$1.1 million less than
54 the next lowest bid, and the highest technical score. This highest technical score was comprised of
55 considerations of aesthetics, maintainability, and long-term durability. As shown in Figure 2, the
56 identical knuckles with curved flange plates and the simple splice connection with fewer fasteners
57 provide a more elegant solution compared to conventional gusset plates. The wide flange members
58 oriented in their strong axis bending gives the bridge a stronger appearance even to the inexperi-
59 enced eye. The behavior of Memorial Bridge has been studied extensively. Shahsavari et al. (2019)
60 investigated its behavior under live load through field monitoring using accelerometers, uniaxial
61 strain gauges, strain rosettes, and tiltmeters. Chen et al. (2018) measured vibrations on the bridge
62 induced by span lift and traffic loads through camera-based field monitoring. A fatigue evaluation
63 has been conducted by Bell and Medina (2019) through experimentally testing a scaled model of
64 the knuckle subjected to cyclic loads. The results indicated that, with no initial imperfections, an
65 infinite fatigue life is expected. Mashayekhi and Santini-Bell (2019) also investigated the fatigue
66 performance of the knuckle through both field measured data and numerical modeling.

67 While the “gussetless” Memorial Bridge addressed the deficiencies of trusses, it is not a modu-
68 lar system. It is a one-of-a-kind structure, not a “kit-of-parts” that can be readily adapted for a wide
69 array of spans and loads. The members in the Memorial Bridge were designed to minimize the
70 number of field connections along the span. This resulted in knuckle and chord members (which
71 are built-up steel sections) being fabricated as a single piece, all of which were sufficiently large to
72 require crane erection. In contrast, modular systems should be sufficiently small to fit in shipping
73 containers and to minimize erection equipment requirements. The research in this paper is inspired
74 by Memorial Bridge, but addresses the prime deficiency of that one-of-a-kind structure: that it is
75 not modular.

76 This research introduces a new strategy for the accelerated fabrication and erection of steel
77 bridges: a modular joint (Figure 4). The modular joint is comprised of a weldment/built up section
78 of web and continuous flanges that includes a starter segment for connection to other modular joints
79 or wide flange structural members. Like the knuckle of Memorial Bridge, the joint can be easily
80 fabricated as the webs and flanges are cut from flat plate. Flanges are cold bent and then welded to
81 the web. Unlike Memorial Bridge, its dimensions are selected for transportability in shipping con-
82 tainers standardized by the International Standard Organization (ISO containers, hereafter). Mod-
83 ular joints can be joined to one another through conventional splice connections to form short-span
84 structures (Figure 5A). Longer spans and increased capacity can be achieved by using standard
85 rolled wide flange members as both the chords and diagonals, all also joined through conventional
86 splice connections (Figure 5B). The wide flange members are easily acquired and only require
87 that holes be drilled. Flanges and webs are connected independently through double shear, bolted
88 splice connections, forming a moment-resisting connection (Figure 6A). This provides flexural
89 stiffness for truss-like or beam-like behavior, providing the potential for the structure to tolerate
90 member loss for enhanced resiliency. Variability in depth (Figure 5C), for improved efficiency or
91 site constraints, is achieved by changing the length of the members and using bent flange splice
92 plates. These bent splice plates can be prebent to a desired angle (e.g., by a press brake) or the
93 adjustable bolted steel plate connection can be used (Figures 6B and 6C).

94 The adjustable bolted steel plate connection is a slip-critical, angled splice connection com-
95 prised of flange plates that are cold bent by press brake to a set of angles. Adjustability is achieved
96 by further cold bending the prebent plates in the field through bolt tightening to achieve the desired
97 angle (Gerbo et al., 2016b, 2018, 2020a,b). The bend radius of the plates (even with field bending
98 via bolt tightening) is required to exceed five times the thickness of the splice plate, in accordance
99 with the current bridge construction code (AASHTO, 2017a). This limit is based on the study by
100 Keating and Christian (2012) which found that strains in plates induced by cold bending should not
101 exceed 10 %, as higher strains would reduce the fracture toughness and ductility of the steel. The
102 service and ultimate (bolt shear failure) behavior of the adjustable bolted steel plate connection

103 was experimentally tested by Gerbo et al (2020b). This research found that the slip and ultimate
104 bolt shear capacity of the adjustable bolted steel plate connection can be reduced (depending on
105 the geometry) due to reduced clamping loads and lack of engagement of shear planes, respectively.
106 Recommendations for the calculation of these reduced capacities were developed. Additional re-
107 search is necessary for implementation of the adjustable bolted steel plate connection.

108 The approach proposed in this paper is modular, because identical joints are used repeatedly
109 throughout the structure and among many structures. It is a kit-of-parts - comprised of (1) modular
110 joints that are transportable in an ISO container, (2) standard rolled wide flange sections as both
111 chords and diagonals, and (3) bolted splice connections - that can be used for a wide variety
112 of span lengths and loadings. In comparison to existing modular systems, it can achieve longer
113 spans [targeting 119 m (390 ft)], while providing capability for shorter 39.6-m (130-ft) spans. A
114 more efficient truss topology carries load primarily through axial tension and compression, with
115 the adjustable connection enabling varying depth, to change profile with demand. Bolted splice
116 connections, as opposed to pins, allows for a more durable and reliable connection. The use of
117 double shear connections as opposed to single shear (as is typically used in conventional gusset
118 plates) results in a fewer number of fasteners, thereby reducing assemblage time and facilitating
119 the erection process. This approach also eliminates any field welding thus, saving cost and time.
120 As the modular joint is inspired by the Memorial Bridge, bridges developed using the proposed
121 approach would be able to achieve similar aesthetic qualities. A key aspect to truss performance
122 is joint behavior, and the development of new, more robust joints combined with the principles
123 that have made modular bridges so successful, offers a new paradigm in modular construction: the
124 joint is modularized.

125 **OBJECTIVES AND SCOPE**

126 The objective of this research is to develop a modular joint for the accelerated fabrication and
127 erection of steel bridges and to numerically investigate the behavior of this joint. A methodology
128 is developed to achieve rational constant- and variable-depth bridge forms within the constraints of
129 the developed kit-of-parts. Sizing optimization of the modular joint is performed to minimize the

130 structure self-weight while meeting structural performance demands and transportability criteria.
131 The promise of the modular joint is demonstrated through finite element (FE) numerical analy-
132 ses. Ultimately, this research presents a fundamentally new approach to, modular construction in
133 which the joint becomes the module and wide flange sections are used to achieve varying structural
134 geometries through bolted splice connections.

135 **DEVELOPMENT OF BRIDGE FORMS**

136 **Geometric Parameters of the Modular Joint**

137 To develop a modular joint for both constant- and variable-depth forms, the following geomet-
138 ric parameters are defined (Figure 7): (1) joint length, l , (2) joint angle, θ between the chords and
139 diagonals, (3) depth of the web, h , (4) radii of curvature, R_1 and R_2 of the bent flanges, (5) starter
140 segments lengths, d_1 and d_2 , and (6) thickness of the flanges, t_1 , t_2 , and t_3 and the web, t_4 . In this
141 research, it is envisioned that a single modular joint (symmetric about a vertical centerline at point
142 O) would be used throughout a structure and among many structures, to take advantage of cost
143 and time savings of mass producing identical components. However, there could be advantages
144 in considering several versions of the modular joint. For example, smaller and lighter modular
145 joints could be used for shorter span or lower capacity structures. Asymmetric joints or joints with
146 different angles, θ could be considered for different geometries. These are potential areas of future
147 research.

148 As the geometry is determined such that a single modular joint can be used for many spans,
149 the joint length, l is chosen to be 3.05 m (10 ft). This is consistent with the length of the panels
150 of existing modular systems, indicating that the joints could be readily handled. The floor beam
151 spacing, for all spans, is also chosen to be this length. For the back-to-back joint layout for short
152 spans, the floor beams would only connect to the modular joints. For longer spans, the lower chord
153 joint spacing is then chosen to be integer multiples of this length, with the floor beams between
154 joints connected to the lower chord. For constant-depth bridges, this is also the upper chord joint
155 spacing. The floor beams which are standard rolled wide flange sections are connected to the
156 lower chord members or lower chord modular joints through web stiffeners using double shear

157 splice plate connections. The bottom flanges of the floor beams are connected to the flat flange
158 of the lower chord modular joints or to the bottom flange of lower chord members through single
159 shear splice plate connections. A lateral bracing system, including portal bracing, also comprised
160 of standard wide flange sections, would be developed for longer span bridges. Lateral bracing
161 would not be provided for the bridge with back-to-back joints layout due to traffic interference.
162 The lateral bracing system is connected to the flat flange of the upper chord joints via plates.
163 Single angle shear connections connect the web of the lateral bracing to web stiffeners (at each
164 upper chord modular joint) as well as to the web of the modular joints. All of these components
165 would be standardized within the kit-of-parts.

166 The joint angle, θ is chosen to be 60° so that the lengths from the center of the joint to the
167 end of each starter segment are equal, ensuring a compact shape for transportation. For longer
168 spans, wide flange members will be utilized between modular joints. As a function of the rolling
169 process, the web depth (i.e., the total section depth minus the thickness of the two flanges) of wide
170 flange sections is approximately the same for sections with the same WXX designation, where XX
171 refers to the nominal depth. This research considers wide flange sections from W14x109 through
172 W14x257 (AISC, 2011), and therefore a joint web depth, h is chosen to be equal to the average
173 web depth of these sections [$h = 320$ mm (12.6 in.)]. Fill plates could also be used for other section
174 sizes.

175 The radii of curvature, R_1 and R_2 , are both selected to be 508 mm (20 in.). There is an ad-
176 vantage of using larger bend radii as this would increase the overall joint depth and therefore also
177 increase the cross sectional area to transfer the loads from the flange to the web. Furthermore,
178 larger bend radii reduce stress concentrations at welds, making it less sensitive to fatigue (Coving-
179 ton et al., 2013). To satisfy bridge construction code requirements (AASHTO, 2017a) based on
180 limiting residual strains that reduce the fracture toughness and ductility of the steel (Keating and
181 Christian, 2012), the radii must also exceed five times the thickness of the flanges. In this research,

182 Grade 50 structural steel is used for the modular joint. The starter segments lengths, d_1 and d_2 are:

$$d_1 = \frac{l}{2} - \frac{\frac{h}{2} + R_1}{\tan \frac{\theta}{2}}; \quad d_2 = \frac{l}{2} - \frac{\frac{h}{2} + R_2}{\tan \frac{\theta}{2}} \quad (1)$$

183 The joint is designed to be nested in ISO containers together with wide flange members, for
184 transportation as shown in Figure 8. Rows of these nested modular joints and members could be
185 arranged on racks that can be offloaded by forklift or other lifting equipment. The ISO container
186 considered for this research has an inner length, $E = 12$ m (39 ft 4 in.) and inner depth, $I = 2.67$ m
187 (8 ft 9 in.) (ISO, 2013).

188 Sizing optimization will be used to determine the thicknesses of the joint flanges and web, as
189 well as the section size of the wide flange members.

190 **Forms for Constant-Depth Bridge**

191 This research develops a “family” of constant-depth simply supported bridges using the mod-
192 ular joint with different spans, S (Figure 9). The focus is on achieving a span of $S_1 = 119$ m (390
193 ft) that is the longest in the family and exceeds the span limitations of existing modular systems,
194 while also providing capabilities for shorter spans: $S_2 = 79.2$ m (260 ft) and $S_3 = 39.6$ m (130
195 ft). For each, the joint spacing is bl , where b = number of floor beams between two successive
196 modular joints + 1, as this would facilitate floor beams spaced at a distance, l apart. The depth, D
197 is: $D = \frac{bl}{2} \tan \theta$. For each of these 3 spans, the span-to-depth ratio is 15, which is efficient and
198 economic for truss bridges. The number of modular joints for each is also the same (i.e., 27), such
199 that the kit-of-parts would require the same number of joints regardless of the span. The modular
200 joint would be designed for the highest demand and could then be used for the entire family.

201 **Forms for Variable-Depth Bridges**

202 Variable-depth bridges, in which modular joints are connected to wide flange members at vary-
203 ing angles, γ , can be achieved using conventional bent connections or the adjustable bolted steel
204 plate connection (Figure 5C, 6B and 6C) (Gerbo et al., 2016b, 2018, 2020a,b). This can facilitate
205 more efficient structural forms as depth can be varied with demand and/or accommodate site or

206 architectural constraints. In this configuration, the ends of the members would be cut to the desired
207 angle prior to erection. With the aim of maintaining uniform modular joints throughout the struc-
208 ture, the ends of the starter segments of the joints would not be cut. Thus, a uniform gap between
209 members and joints can be achieved without sacrificing modularity (Figure 6B).

210 This research proposes a methodology for developing variable-depth bridges based on a struc-
211 tural performance metric for a given (1) span length, S , (2) span type (i.e., simply supported or
212 three-span continuous), and (3) desired variable-depth shape with a prescribed depth, D at the
213 abutments. For varying values of peak depth, H (at midspan for the simply supported, at the in-
214 terior supports for the continuous) and largest magnitude of angle between components, γ_{max} , the
215 desired variable-depth shape is defined, the coordinates of the modular joints are found, and the
216 structural performance metric is evaluated. The methodology develops a set of solutions for which
217 the structural performance can be readily compared and a designer can select a variable-depth
218 form. This methodology determines the angles between the modular joints and the members, γ , as
219 well as the length of the members.

220 The following formulation assumes through-type bridges with variable depth upper chords and
221 lateral bracing provided at each upper chord joint. The lower chord joints are assumed to be flat
222 and spaced at a fixed spacing, bl to be consistent with the uniform floor beam spacing discussed
223 earlier. Other configurations could be considered using an analogous methodology.

224 The structural performance metric in this research is related to reducing the susceptibility to
225 member buckling of upper chord, lower chord, or diagonals members that are in compression under
226 any of the load cases considered. This metric was chosen as member buckling is a major factor in
227 the design of truss bridges. This is quantified as the highest magnitude FL^2 (related to Euler buck-
228 ling) for any compressive member in the structure, where F is the force in the member calculated
229 using the direct stiffness method (DSM) and L is the unbraced length of the member. Analysis
230 using the DSM models the members as two-node frame elements that are moment-connected (as
231 the modular joint can carry flexure). The modular joints are not modeled explicitly. The unbraced
232 length, L used in this metric is the length of the frame element connecting the center points O of

233 the two joints, as it is assumed that lateral bracing or floor beams will be provided at each joint.
234 The kinks formed between the modular joint and wide flange members were not considered. The
235 effect of the kinks on behavior could be evaluated through high-fidelity numerical models, which
236 would be time-consuming to build and analyze for a large number of considered geometries. The
237 focus of this paper is on developing a quick and computationally inexpensive way to compare the
238 behavior of many different bridge forms. A uniform section size for all members was assumed
239 (i.e., W14x109) for simplicity. The load includes self-weight of the upper chord, lower chord, and
240 diagonal members (applied as uniformly distributed load along each member, neglecting the joints
241 for simplicity), self-weight of a lightweight deck of 1.2 kN/m^2 (25 psf), and two lanes of vehicular
242 traffic represented by two design lane loads from the bridge design code [18.7 kN/m (1.28 kips/ft)
243 in total (AASHTO, 2017b)]. The live load and the load from the lightweight deck are applied as
244 a uniformly distributed load at the lower chord along the entire span, as the floor beams are con-
245 nected to the lower chord members at every $l = 3.05 \text{ m}$ (10 ft) in the longitudinal direction. Note
246 that this is different from typical trusses where the load is transferred only at the nodes. Only one
247 plane of the bridge is modeled. For the simply supported bridge, the boundary conditions are: at
248 one end, free rotation about the transverse axis, translation restrained in longitudinal and vertical
249 directions; at the other end, free rotation about the transverse axis, free translation along the lon-
250 gitudinal axis, and translation restrained in the vertical direction. For the continuous bridge, the
251 boundary conditions are: at one of the abutments, free rotation about the transverse axis, translation
252 restrained in longitudinal and vertical directions; at the piers and the other abutment, free rotation
253 about the transverse axis, free translation along the longitudinal axis, and translation restrained in
254 the vertical directions. When comparing different bridge forms, the bridge with the lowest value
255 of the structural performance metric would have the lowest susceptibility to member buckling and
256 would, therefore, be preferred. Other performance metrics could be considered. This research
257 selects the structural performance metric, FL^2 , as it allows the methodology to rapidly search the
258 design space for finding a bridge form with an enhanced member buckling capacity.

259 To achieve the variability in depth, this research utilizes the adjustable bolted steel plate con-

260 nection. The connection is comprised of 10° , 20° , and 30° prebent flange plates, as well as flat
 261 plates, that can be further bent in the field through bolt tightening to achieve the desired angles. To
 262 limit the strains induced in the bolts during field installation, the amount of field bending should
 263 be limited to $\pm 5^\circ$ (Gerbo et al., 2018, 2020a,b). As only angle changes up to $\gamma = 35^\circ$ were investi-
 264 gated, this research considers angle changes, γ between components up to this value. As there are
 265 benefits in smaller angles and in using flat plates that are field-bent only, this research considers
 266 differing values of the largest magnitude of allowed angle, γ_{max} . Conventional, bent splice plate
 267 connections could alternatively be used.

268 The desired variable-depth shape of each bridge can be defined based on structural demands,
 269 site constraints, or other priorities. In this paper, the desired shape for the simply supported bridge
 270 is a parabola, with a depth, D at the abutments and a depth, H at midspan ($H > D$), to approximate
 271 the bending moment diagram of a simply supported beam under uniformly distributed load. For
 272 the three-span continuous bridge, the desired shape in this research relates to the envelope of the
 273 moment diagram with a uniformly distributed load over: (1) the entire bridge span, (2) half of the
 274 entire bridge, (3) on any of the three spans, and (4) on any of the two spans. For each load, the
 275 highest value of the moment is calculated and is scaled to relate to a depth, H at the inner supports.
 276 At the abutments, the height is the depth, D .

277 The coordinates of the upper chord joints, U_i (where the index i is counted from the origin Ω ,
 278 Figure 10) and the lower chord joints, L_i are found to achieve the variable-depth shape. As shown
 279 in Figure 10, the angles between the modular joints and the members, $\gamma_i^1 - \gamma_i^6$ (Table 1), vary to
 280 achieve the variable depth form. Specifically, the coordinates of the upper chord joints are found by
 281 varying the angles γ_i^1 , γ_i^2 , and γ_{i-1}^4 between $-\gamma_{max}$ and $+\gamma_{max}$, with increments of 1° . The value
 282 of γ_i^3 , γ_i^5 , and γ_i^6 are calculated using the equations in Table 1 and discussed later. As the angles
 283 vary, thousands of different locations for each upper chord modular joint exist. The permutation
 284 of these angles that gives coordinates for each upper chord joint closest to the desired depths, y
 285 at a distance, x from the origin Ω are selected. In this methodology, the angles, γ_i^3 , γ_i^5 , and γ_i^6
 286 are required to be less than or equal to γ_{max} and the length of each upper chord, T_i , diagonal, N_i

287 and M_i , and lower chord, G_i member is required to be less than the inner length, E of the ISO
 288 shipping container (Figure 8). If these criteria are not met for a joint, the permutation of angles is
 289 rejected and the next permutation with coordinates closest to the desired shape, that also satisfies
 290 the length criteria, is selected. Following this methodology, the coordinates of the upper chord
 291 joints are found progressively moving out from the origin to ultimately achieve the desired span
 292 length. Both the simply supported and continuous bridges are assumed to be symmetric about their
 293 center lines.

294 *Simply Supported Bridges*

295 The coordinates of the joints (where coordinate of a joint refers to the location of the joint
 296 center O) for the simply supported bridge are found by beginning with an upper chord joint, U_0
 297 placed at midspan (i.e., $x_{U_0} = 0$ and $y_{U_0} = H$), as its corresponding lateral brace would restrain
 298 the system against buckling at this location of highest compression (Figure 10A). It is parallel to
 299 the lower chord for symmetry. The coordinates of joint L_0 are: $x_{L_0} = bl/2$ and $y_{L_0} = 0$, with the
 300 coordinates of the subsequent lower chord joints L_i (for $i > 0$) being:

$$x_{L_i} = x_{L_{i-1}} + bl \quad (2)$$

$$y_{L_i} = 0 \quad (3)$$

301 The coordinates of the subsequent upper chord joints U_i (for $i > 0$) are:

$$x_{U_i} = x_{L_{i-1}} + (l/2) \cos \theta + (l/2) \cos(\theta - \gamma_i^1 + \gamma_i^5) + M_i \cos(\theta - \gamma_i^1) \quad (4)$$

$$y_{U_i} = (l/2) \sin \theta + (l/2) \sin(\theta - \gamma_i^1 + \gamma_i^5) + M_i \sin(\theta - \gamma_i^1) \quad (5)$$

302 where γ_i^1 is varied between $-\gamma_{max}$ and $+\gamma_{max}$ and $\gamma_i^5 = \gamma_i^1 - \alpha_i$. The angle, $\alpha_i = \alpha_{i-1} + \gamma_i^2 + \gamma_{i-1}^4$,
 303 is between OA' of joint U_i and a horizontal line passing through the center O of joint U_i . Note that

304 $\alpha_0 = 0$. Angles γ_i^2 and γ_{i-1}^4 are varied between $-\gamma_{max}$ and $+\gamma_{max}$. M_i is the length of the diagonal
 305 member center line and is calculated as follows:

$$M_i = f \frac{\sin \eta}{\sin \psi} - u \frac{\sin \varphi}{\sin \psi} \quad (6)$$

306 where $f = \sqrt{\Delta e^2 + \Delta v^2}$ with:

$$\Delta e = x_{L_{i-1}} - x_{U_{i-1}} + (l/2)(\cos \theta - \cos \alpha_{i-1}) \quad (7)$$

$$\Delta v = y_{U_{i-1}} - (l/2)(\sin \theta + \sin \alpha_{i-1}) \quad (8)$$

307 The distance, $u = l \cos \theta$ is determined from triangle $B'O'A'$. The angles are: $\psi = \theta - \gamma_i^1 +$
 308 $\gamma_{i-1}^4 + \alpha_{i-1}$, between the upper chord T_{i-1} and the diagonal M_i , $\eta = \beta - \gamma_{i-1}^4 - \alpha_{i-1}$, between
 309 the upper cord T_{i-1} and line CA , and $\varphi = 90 - (\theta/2) + \gamma_i^2$, between $A'P$ and $B'A'$. The angle,
 310 $\beta = \arctan(\Delta v/\Delta e)$ is between the horizontal and line CA . Once the coordinates of U_i and L_i
 311 are found, the angles γ_i^3 and γ_i^6 need to be calculated to ensure that they are less than γ_{max} :

$$\gamma_i^3 = \gamma_i^6 - \gamma_i^1 + \gamma_i^5 \quad (9)$$

312

$$\gamma_i^6 = \arctan \left(\frac{y_{U_i} - (l/2) [\sin \theta + \sin(\theta + \alpha_i)]}{x_{L_i} - x_{U_i} - (l/2) [\cos \theta + \cos(\theta + \alpha_i)]} \right) - \theta \quad (10)$$

313 The lengths of each member is calculated to ensure that they are less than E . The upper chord T_{i-1}
 314 is:

$$T_{i-1} = f \frac{\sin(\psi + \eta)}{\sin \psi} - u \frac{\sin(\psi + \varphi)}{\sin \psi} \quad (11)$$

315 The lower chord members have a constant length due to fixed lower chord spacing: $G_i = (b - 1)l$.

316 The length of the diagonal member M_i is given by Equation 6 and the length of diagonal member

317 N_i is:

$$N_i = \frac{y_{U_i} - (l/2) [\sin \theta + \sin(\theta + \alpha_i)]}{\sin(\theta + \gamma_i^6)} \quad (12)$$

318 Because the angles γ must not exceed γ_{max} , there is a limit on the depth at midspan, H_{max} which
 319 can be calculated as follows:

$$H_{max} = l \sin \theta + [(bl/2) - l \cos \theta] \tan (\theta + \gamma_{0,max}^6) \quad (13)$$

320 where $\gamma_{0,max}^6$ is equal to γ_{max} .

321 These equations do not account for interference between the joints and members which can be
 322 avoided by cutting members or decreasing d_1 and d_2 . These equations assume a clockwise angle
 323 is negative, and a counterclockwise angle is positive.

324 *Three-span Continuous Bridges*

325 The coordinates of the continuous bridge can be similarly found, with the following differences.
 326 A lower chord joint, L_0 , is positioned above the first pier, which serves as the origin, Ω (i.e., $x_{L_0} =$
 327 0 ; $y_{L_0} = 0$, Figure 10B). The upper chord, T_0 is centered above this joint and oriented horizontally
 328 at the depth, H . Thus, $\gamma_4^0 = 0$ and $\alpha_0 = 0$. The coordinates of the first upper chord joint, U_1 are
 329 calculated using Equations 2 through 8, with the exception that for this joint, $\Delta e = (l/2) \cos \theta$ and
 330 $\Delta v = H - (l/2) \sin \theta$. The coordinates of the successive modular joints, as well as the equations
 331 for the angles, γ and the member lengths, are the same as the prior section.

332 The limit, H_{max} on the depth at the piers is found by varying the angles γ_1^1 , γ_2^1 and γ_6^1 between
 333 $-\gamma_{max}$ and γ_{max} . For each permutation of these angles, a depth H_s is calculated: $H_s = y_{U_1} +$
 334 $(l/2) \sin \alpha_1$, where y_{U_1} is calculated using Equation 5 but with M_1 found as follows:

$$M_1 = \frac{(l/2) \tan(\theta + \gamma_6^1) [2b - 2 \cos \theta - \cos(\theta - \alpha_1) - \cos(\theta + \alpha_1)] + (l/2) [\sin(\theta + \alpha_1) - \sin(\theta - \alpha_1)]}{\sin(\theta - \gamma_1^1) + \cos(\theta - \gamma_1^1) \tan(\theta + \gamma_6^1)} \quad (14)$$

335 Note that Equation 6 cannot be used, as the depth is not yet known. The angles γ_1^3 and γ_1^5 must not
 336 exceed γ_{max} . The largest value of H_s will be H_{max} .

337 **Form Comparison**

338 The proposed methodology is implemented for two case studies: a 119-m (390-ft) simply
339 supported bridge and a three-span continuous bridge [101 m (330 ft) - 119 m (390 ft) - 101 m
340 (330 ft) spans]. For both, $b = 3$, corresponding to a lower chord joint spacing of 9.14 m (30 ft).
341 The depth, D at the abutments is the same depth as the constant-depth 119-m (390-ft) simply
342 supported bridge for comparison [$D = 7.92$ m (26 ft)]. The depth H varies between D and H_{max}
343 in increments of 0.305 m (1 ft). A comparison of the structural performance metric for varying
344 values of H and γ_{max} are shown in Figure 11. The performance metric for constant-depth forms
345 are also included for comparison.

346 The developed methodology is a geometric problem coupled with structural performance crite-
347 ria that develops variable-depth forms within the constraints of the modular joint approach. There
348 is a large number of permutations in the solution space depending on the ranges of the angles γ ,
349 number of modular joints along the bridge span, and length of the members. For example, for the
350 constant-depth 119-m (390-ft) simply supported bridge with $\gamma_{max} = 15^\circ$, there are 29,791 possible
351 permutations of the angles for a single upper chord joint. There are six upper chord joints along
352 half of the span resulting in 178,746 permutations that were investigated for a given depth H . For
353 the $\gamma_{max} = 15^\circ$ case, there are 22 different values of H resulting in a total of 3,932,412 permu-
354 tations. The goal of this study was to explore all possible permutations of the angles γ , while
355 limiting the susceptibility of the compressive members to buckling as well as limiting the length of
356 the wide flange members to meet transportation requirements. These criteria also limit the depth
357 of the structure. Deeper bridges have lower peak compressive forces but increased length of the
358 members. The lowest possible span-to-depth ratio is approximately 8.1 calculated based on the
359 highest value of the depth limit, H_{max} . The highest span-to-depth ratio is 15. The resulting range
360 for the span-to-depth ratio is reasonable for truss bridges. This solution space demonstrates the
361 importance of the methodology to be able to quickly select an efficient form.

362 Figure 11A shows that the lowest value of the structural performance metric for the simply
363 supported bridge corresponds to $\gamma_{max} = 15^\circ$ and $H = 14$ m (46 ft), resulting in a span-to-depth

364 ratio of 8.5 (close to the lowest value). The magnitude of FL^2 depends mainly on the term L^2 .
365 However, as γ_{max} increases and L^2 increases as a result, the magnitude of FL^2 actually decreases.
366 This is primarily because by using larger angles, deeper bridges are developed resulting in lower
367 peak compressive forces that are found in the upper chord members at midspan. Additionally, the
368 upper chord member lengths are limited by the lower chord joint spacing and have approximately
369 constant length along the span of 9.14 m (30 ft) for different γ_{max} . Only the length of the diagonal
370 members changes substantially as γ_{max} increases. However, these members have very small (close
371 to zero) axial forces which ultimately lowers the magnitude of FL^2 . Thus, as expected, deeper
372 bridges are less susceptible to member buckling. The relative difference in FL^2 between the
373 constant-depth bridge and the lowest FL^2 variable-depth bridge is 40% which shows the value of
374 considering variable-depth forms using the modular joint.

375 Figure 11B shows that the lowest value of the structural performance metric for the continuous
376 bridge corresponds to $\gamma_{max} = 35^\circ$ and $H = 10.7$ m (35 ft), resulting in a span-to-depth ratio of
377 11.1. Here, the magnitude of FL^2 is mostly influenced by the diagonal members at the piers
378 which have significant compressive forces and are also the longest members. As a result, the forms
379 corresponding to the lowest magnitude of FL^2 for each γ_{max} have a high span-to-depth ratio and
380 a depth, H less than 12 m (40 ft). Although, the peak compressive forces are in the lower chord
381 at the piers, their unbraced length is 9.14 m (30 ft) and therefore, did not significantly impact
382 the results. As in the simply supported case, a variable-depth upper chord reduces the magnitude
383 FL^2 compared to the constant-depth bridge. However, the relative difference in FL^2 between
384 the constant-depth bridge and the lowest FL^2 variable-depth bridge is just 18%. This is mostly
385 because the two bridges have similar forms and thus, similar susceptibility to member buckling.

386 **STRUCTURAL OPTIMIZATION**

387 A sizing optimization approach for minimum self weight, with the aim of improving material
388 efficiency and ease of transportation and erection while meeting geometric and structural con-
389 straints, is proposed. This is implemented for the two lane, 119-m (390-ft) long constant-depth
390 simply supported bridge, thereby optimizing the joint for the highest demand of the constant-depth

391 forms. The constant-depth (as opposed to variable-depth) bridge is selected as it has higher suscep-
 392 tibility to member buckling and therefore also represents a worst case scenario. The optimization
 393 approach could also be used with other span configurations, span lengths, and/or variable depths.
 394 Alternatively, this sizing optimization could be formulated as a multi-objective optimization prob-
 395 lem that includes both weight and fabrication/erection cost. This is a potential area for future
 396 research.

397 **Problem Formulation**

398 The formal sizing optimization problem formulation is as follows:

$$\begin{aligned}
 & \underset{\mathbf{t}, \mathbf{s}}{\text{minimize}} && W(\mathbf{t}, \mathbf{s}) = aJ(\mathbf{t}) + pV(\mathbf{s}) + rQ(\mathbf{s}) + oZ(\mathbf{s}) \\
 & \text{such that:} && c_1 = 5t_k - R_k \leq 0; \quad k = 1, 2 \\
 & && c_2 = 3g + w + \frac{h + t_1 + t_3}{2}(4 + \cos \theta) + \frac{l}{2} \sin \theta - I \leq 0 \\
 & && c_3 = \sigma - \sigma_N \leq 0; \\
 & && c_4 = F_D + \xi F_L - \lambda F_L \leq 0; \\
 & && c_5 = 0.8K_{in} - K_p \leq 0; \\
 & && c_6 = \tau_m - \tau_n \leq 0; \\
 & && \mathbf{t} \in \mathbf{S}_T; \quad \mathbf{S}_T \in [12.7, 15.9, \dots, 63.5] \text{ mm} \\
 & && \mathbf{s} \in \mathbf{S}_B; \quad \mathbf{S}_B \in [W14x109, W14x120, \dots, W14x257]
 \end{aligned} \tag{15}$$

399 The design variables, \mathbf{t} refer to the thickness of the bent flange plates, t_1 and t_2 , flat flange plate,
 400 t_3 , and web plate, t_4 (Figure 7), which are selected from a discrete set, \mathbf{S}_T in the range between
 401 12.7 mm (0.5 in.) and 63.5 mm (2.5 in.) with an increment of 3.175 mm (0.125 in.). All joints are
 402 assumed to be the same. The design variables, \mathbf{s} refer to lower chord, upper chord, and diagonal
 403 member section sizes, s_1 , s_2 , and s_3 , respectively, which are selected from a discrete set, \mathbf{S}_B of ten
 404 different W14 standard wide flange sections in the range between W14x109 and W14x257 (AISC,
 405 2011). Each member type is assigned the same design variable to simplify fabrication and erection

406 (e.g., all upper chords are one section).

407 The objective function is to minimize the self-weight, W which is the summation of the weight
408 of a number of joints with weight J , p number of lower chord members with weight V , r number
409 of upper chord members with weight Q , and o number of diagonal members with weight Z . The
410 self-weight is calculated for a single bridge plane, assuming symmetry.

411 **Constraints**

412 The constraints, c are related to (1) limiting strains from cold bending of the flange plates,
413 (2) transportation requirements, (3) fatigue design requirements, (4) global buckling, (5) ultimate
414 behavior under factored load combinations, and (6) global failure mechanism. To evaluate the
415 structural constraints, a parametric FE model was used.

416 Constraint c_1 limits the strains in the bent flange plates induced during cold bending by requir-
417 ing that the bend radius be at least five times the thickness, consistent with bridge construction
418 code (AASHTO, 2017a). Because both radii of curvature are 508 mm (20 in.), constraint c_1 is
419 satisfied for every value in the set, S_T . This constraint is included for completeness.

420 Constraint c_2 limits the size of the joint and members to be sufficiently small for transportation
421 in an ISO container. To achieve the stacking configuration shown in Figure 8, constraint c_2 requires
422 that the combined height of joints and wide flange section does not exceed the internal height of
423 the container, I . A gap [$g = 12.7$ mm (0.5 in.)] is assumed between each surface. The dimension
424 w is the maximum of the depths of the s_1 , s_2 , and s_3 section sizes to be able to accommodate any
425 of the members.

426 Constraint c_3 limits load-induced fatigue cracks in the joints. It is satisfied if the peak von
427 Mises stress, σ , determined from linear elastic analysis, in any of the lower chord joints under
428 the Fatigue I limit state does not exceed the nominal fatigue resistance σ_N . Thus, this constraint
429 requires that the modular joint is classified as having an infinite fatigue life in accordance with
430 the bridge design code (AASHTO, 2017b). Specifically, the joint falls within Category B detail
431 (welded connections including built-up sections with fillet welds or full penetration groove welds
432 that are loaded longitudinally), for which the nominal fatigue resistance, σ_N is 110 MPa (16 ksi).

433 The bridge design code Fatigue I limit state includes a single design truck positioned along the
434 span to produce the worst effect. A dynamic load allowance of 15% is also applied to the design
435 truck load (AASHTO, 2017b). In the FE model, the truck load is represented as point loads applied
436 at the top flange of the floor beams and is positioned along the span to produce the worst effect.

437 Constraint c_4 requires that the structure does not experience instability under dead, uniform
438 live, and wind loads with load factors of 1.25, 1.75, and 1.0, respectively, defined based on the
439 bridge design code Strength V limit state (AASHTO, 2017b). A linear eigenvalue buckling analysis
440 is performed to evaluate constraint c_4 , in which F_D is the dead load, F_L is combined live and
441 wind loads, ξ is the critical buckling load factor determined from the linear eigenvalue buckling
442 analysis by applying live and wind loads on the deflected shape of the structure from dead load
443 (determined from linear elastic analysis), and λ is the minimum acceptable critical buckling load
444 factor ($\lambda=1.5$). The dead load includes a lightweight deck of 1.2 kN/m² (25 psf) as well as the
445 self-weight of all structural steel components. The live load consists of a 18.7 kN/m (1.28 kips/ft)
446 uniformly distributed load to represent two vehicular design lanes. The wind load consists of two
447 design wind pressures, $P_z^W = 1.57$ kN/m² (32.8 psf) and $P_z^L = 0.785$ kN/m² (16.4 psf), respectively
448 applied to the windward and leeward sides, with magnitudes calculated based on bridge design
449 code (AASHTO, 2017b). In the FE model, the self-weight of the deck and the lane load are
450 applied as a pressure along the length of each floor beam acting at the top flange of the beam.
451 The self-weight of all members is applied through the specified density of steel and acceleration of
452 gravity. The wind load is applied as pressure on the web of the lower chord, upper chord, diagonal
453 members, and modular joints in both bridge planes in the transverse direction.

454 Constraint c_5 requires that the structure has a sufficient capacity to sustain dead and live loads
455 with load factors of 1.25 and 1.75, respectively that are defined based on the bridge design code
456 Strength I limit state (AASHTO, 2017b). This is quantified as requiring that at least 80% of the
457 structure's initial elastic stiffness, K_{in} is maintained at this applied load. This is a conservative
458 approximation of a limit state analysis, providing an estimation of the ultimate capacity of the
459 system. The stiffness is calculated based on a load-displacement curve obtained from a linear

460 inelastic FE analysis. Specifically, the stiffness is calculated as the slope between two successive
461 increments in the load-displacement curve:

$$K^{h,h+1} = \frac{P_{h+1} - P_h}{\delta_{h+1} - \delta_h} \quad (16)$$

462 where P is the applied load, δ is the displacement of the lower chord at midspan, and h refers to the
463 increment. The initial elastic stiffness, K_{in} is taken as the average of all stiffness values up until the
464 end of the linear-elastic region. The end of the linear-elastic region is taken as the point where the
465 difference between the values of two successive slopes exceeds 0.1 %. The tangent stiffness, K_p is
466 calculated using Equation 16 for every increment following this point. In the FE analyses, the dead
467 load includes a lightweight deck of 1.25 kN/m² (25 psf) as well as the self-weight of all structural
468 steel components. The live load consists of two lanes of vehicular traffic corresponding to a 18.7
469 kN/m (1.28 kips/ft) uniformly distributed design lane load and two design trucks, positioned to
470 produce the worst effect. A dynamic load allowance factor of 33% is applied to the truck loads
471 (AASHTO, 2017b). In the FE model, the loads are applied as in constraints c_3 and c_4 .

472 Constraint c_6 relates to the failure mechanism of the structure, ensuring that the upper chord,
473 lower chord, or diagonal members fail prior to any of the modular joints as this would be a more
474 desirable and less catastrophic failure mechanism. It is only evaluated if constraint c_5 is satisfied,
475 meaning that the structure maintains more than 80% of the initial elastic stiffness under the Strength
476 I limit state. The failure mechanism is evaluated by considering the deformations of the members
477 and the joints when the structure is progressively overloaded by increasing the uniform live load
478 until $K_p = 80\%K_{in}$. At the node of highest deflection (node j) for the lower chord member
479 (subscript n) and the lower chord joint (subscript m), the difference between two successive nodal
480 rotations along the center line of the bottom flange, τ is calculated as follows:

$$\tau = |\rho_{j-1} - \rho_{j+1}| \quad (17)$$

481 where ρ refers to the nodal rotation and is calculated as follows:

$$\rho_{j-1} = \frac{\delta_j - \delta_{j-1}}{x_j - x_{j-1}} \quad (18)$$

$$\rho_{j+1} = \frac{\delta_j - \delta_{j+1}}{x_j - x_{j+1}} \quad (19)$$

482 where δ is the nodal vertical displacement and x is the nodal coordinate along the longitudinal axis.

483 **3D Parametric Finite Element Model**

484 To evaluate the structural constraints, a three-dimensional parametric FE model was developed
 485 in ABAQUS/Standard (ABAQUS, 2016). The model includes modular joints, chord and diagonal
 486 members, floor beams (section size W14x159), lateral bracing (section size W14x132), and portal
 487 bracing (section size W14x132). Web stiffeners [38.1 mm (1.5 in.) thick] are provided at the lower
 488 chord members, lower chord modular joints, and upper chord modular joints. Plates [25.4 mm
 489 (1 in.) thick] for connecting the lateral bracing to the upper chord are also included in the model
 490 (Figure 12).

491 The components are modeled with S4R or S3R (4 or 3-node reduced integration) shell elements
 492 with six degrees of freedom per node. A mesh refinement study was performed, resulting in a mesh
 493 size of 38.1 mm (1.5 in.) for the joints, both ends of the members, lateral bracing, and floor beams,
 494 where these members connect to the joints, connection plates, and stiffeners. A larger mesh size
 495 of 101.6 mm (4 in.) is used elsewhere to reduce computational expense.

496 The connection between members is represented through the surface-to-surface or node-to-
 497 surface tie constraints which constrain all degrees of freedom. The boundary conditions are: at
 498 one end of plane one, all translation is restrained; at the same end of plane two, free translation
 499 transversely, translation restrained in longitudinal and vertical directions; at the other end of plane
 500 one, free translation longitudinally, translation restrained in transverse and vertical directions; at
 501 the same end of plane two, free translation along the longitudinal and transverse directions, trans-
 502 lation restrained in vertical direction. Free rotation about the longitudinal, transverse and vertical
 503 axes is allowed at all locations. The boundary conditions are applied at the middle of the bottom

504 flange of each of the end joints. For all members, Grade 50 structural steel with specified minimum
505 yield strength 345 MPa (50 ksi), modulus of elasticity 200 GPa (29000 ksi), steel density of 7850
506 kg/m³ (490 lbs/ft³), and Poisson's ratio 0.3 is used. For the linear inelastic analysis, a non-linear
507 material model was developed by prescribing an elastic-perfectly plastic stress-strain relationship.

508 Algorithms

509 The optimization problem (Equation 15) is solved using the stochastic algorithms Simulated
510 Annealing (SA) and Descent Local Search (DLS) that search the design space, as the design vari-
511 ables are discrete and the constraints are nonlinear.

512 SA was first proposed by Kirkpatrick et al. (1983) and has been widely used in structural
513 optimization applications (e.g., Paya et al., 2008; Thrall et al., 2012, 2014; Russell et al., 2014).
514 The method is based on an analogy to the process of forming crystals through heating and slow
515 cooling of a material. At high temperatures, the atoms are able to move randomly, forming new
516 configurations with primarily lower internal energy. However, a certain probability exists that
517 states with higher energy are formed, ultimately allowing the atoms to reach a state with lower
518 energy. This probability, P_r is given by: $P_r = \exp(-\Delta E/T)$, where ΔE is the difference in
519 energy between two configurations and T is the temperature. As the temperature is decreased,
520 the probability of forming states with higher energy also decreases (Arora, 2004; Arora et al.,
521 1994). Based on this concept, Kirkpatrick et al. (1983) introduced an iterative approach to solving
522 optimization problems, where E is related to the objective function, P_r is related to the probability
523 that solutions with a higher value of the objective function are accepted, and T is a parameter that
524 is initially defined and can be controlled. The probability of accepting solutions with higher value
525 objective function allows the algorithm to escape local minima.

526 The SA algorithm implemented in this research begins by selecting the highest value in the
527 discrete sets S_T and S_B as the initial vector of design variables. This ensures an initial solution
528 that is feasible. This initial solution is both the current (i.e., the solution upon which the algorithm
529 is iterating on) and the best solution (i.e., lowest value of the objective function solution). The
530 algorithm then finds a new solution by randomly perturbing the design variables along the length

531 of the discrete set of design variables, S_T or S_B . The number of variables to be varied are randomly
532 selected from 1 to v , where v is a user-defined parameter. The variable(s) to be varied are randomly
533 selected. The variables are perturbed by a random amount up to a user-defined parameter, pm . The
534 direction, up or down the set (S_T and S_B are in ascending order), is randomly selected. The new
535 solution is then evaluated. If the new solution is feasible and the new weight is less than the
536 current weight, the solution is accepted as the current solution. It is also the best solution. If the
537 solution is feasible, but the weight is higher than the current solution, the algorithm calculates the
538 probability of accepting the higher-weight solution as the current solution. If it is not accepted,
539 the solution is rejected and the current solution is maintained. The algorithm continues for a user-
540 defined number of iterations, m which forms one cycle. For each cycle, T is kept constant. For
541 the first cycle, an initial value for T is chosen such that the acceptance of higher value objective
542 functions is between 20% and 40% (Medina, 2001). Due to the huge computational expense to run
543 all structural analyses, the initial T is found without evaluating any of the structural constraints (c_3
544 to c_6). When all iterations within a cycle are finished, the temperature is reduced by a user-defined
545 parameter, rt and a new cycle begins. The reduction of T decreases the probability of accepting
546 higher value solutions. The algorithm converges when for a predefined number of cycles, n the
547 best solution has not been updated.

548 The DLS algorithm is similar to SA, but without the probability of accepting higher objective
549 function value solutions. Convergence is defined as a certain number of iterations, it for which the
550 objective function has not been decreased. The best solution is the last feasible solution.

551 **Results**

552 The results from several tests using the SA and DLS algorithms, including the average solution
553 weight, μ , standard deviation, σ , coefficient of variation, C_v , lowest weight, W_{min} , and corre-
554 sponding design variables, t and s are presented in Table 2. Both SA and DLS algorithms result
555 in nearly equivalent lowest weight solutions [SA-1 and SA-2 with 1201 kN (270 kips) and DLS-1
556 with 1208 kN (272 kips)]. Although, SA finds the lowest weight solution, the lower coefficient of
557 variation for DLS-1 indicates better convergence among the ten tests. The convergence curve of

558 the lowest weight solution for both algorithms is presented in Figure 13. The capability of SA to
559 escape local minimum is clearly shown by the fluctuations in Figure 13A. However, both curves
560 follow a similar trend and ultimately converge to almost the same weight. This shows that both
561 algorithms are suitable for the optimization problem. However, comparing the time to run the opti-
562 mization, DLS is able to achieve convergence more rapidly than SA. This is particularly noticeable
563 when the numbers of cycles, n are increased as in SA-2 which resulted in a significantly higher
564 run time compared to the other cases.

565 The lowest weight solution of $W_{min} = 1201$ kN (270 kips), which is found through both SA-1
566 and SA-2, results in an optimized design of the modular joints and members. The algorithm has
567 selected member section sizes primarily to satisfy the global buckling and failure constraints (c_4
568 and c_6). Due to bending of the lower chord and peak tensile forces at midspan, the lower chord
569 members are W14x233 wide flange sections that have high in-plane stiffness. Similarly, to provide
570 enough compressive capacity in the upper chord and prevent member buckling, the algorithm has
571 selected W14x193 wide flange sections which have a considerable out-of-plane stiffness. Because
572 the diagonal members have very small (close to zero) forces, the algorithm has selected the wide
573 flange section W14x109 which is the smallest in the given database. The modular joint design
574 variables are similarly selected based primarily on the structural constraints. Due to high axial
575 forces in the chords, the thickness of the bent flange plate, t_1 is 50.8 mm (2 in.) which is close
576 to the limit value of 63.5 mm (2.5 in.). This is because a large cross sectional area is required to
577 transfer the chord axial forces from the bent flange to the web zone. The thickness $t_3 = 34.9$ mm
578 (1.375 in.) has a lower value compared to t_1 which is primarily because the flange is a straight plate
579 and provides a continuous load path of the chords axial forces. The small forces in the diagonal
580 members result in a thickness $t_2 = 12.7$ mm (0.5 in.), the smallest value in the database. The
581 algorithm, however, selects a relatively thick web plate, $t_4 = 44.5$ mm (1.75 in.). The thick web
582 provides a way to handle the stress concentration within the web zone. Furthermore, a thicker web
583 plate is beneficial for the global stability of the structure as it increases its out-of-plane stiffness.

584 The response of the lowest weight solution is presented in Figure 14. The load-displacement

585 curve (Figure 14A) developed from a linear inelastic analysis indicates that the structure's response
586 under Strength I limit state (constraint c_5) is primarily in the linear elastic range which is desir-
587 able. The structure can sustain loads up to 12,035 kN while keeping more than 80% of its initial
588 stiffness. As the bridge is overloaded (increased uniform live load), the initial elastic stiffness is
589 reduced which is indicated in the load-displacement curve by a slight plateau range followed by
590 a positive slope. At a load of 12,268 kN, the tangent stiffness, K_p is 80% K_{in} . The structure has
591 developed excessive deformations and, as clearly shown in Figure 14B, the wide flange member
592 in the lower chord at midspan has failed. Figure 14C shows the buckling mode shape of the struc-
593 ture corresponding to the critical load factor $\xi = 3.9$ which is bigger than the minimum acceptable
594 buckling load factor $\lambda = 1.5$. The structure experiences local out-of-plane buckling at midspan
595 where the compressive force is the highest which is also a desirable and expected mode shape.

596 CONCLUSIONS

597 This paper presented a modular joint as a new approach for the accelerated fabrication and erec-
598 tion of steel bridges. It is a kit-of-parts comprised of (1) a single prefabricated modular joint, (2)
599 standard wide flange sections, and (3) bolted splice connections, that can be used for a wide range
600 of spans and loadings. The modular joint provides significant structural advantages including: (1)
601 the use of splice connections in double shear increases the efficiency and decreases construction
602 time and cost by reducing the number of bolts, (2) the splices are located to facilitate inspection,
603 maintenance, and repair, and (3) the strong axis orientation of the members and moment-resisting
604 connections between the members results in increased flexural capacity, providing the potential for
605 the chords to carry load in bending if a diagonal is lost. The modular joint, in comparison with
606 the existing modular systems, can achieve greater spans, while providing capabilities for shorter
607 spans.

608 This paper proposed methodologies for achieving rational-form bridges. More specifically, a
609 family of constant-depth simply supported bridges with different span lengths was introduced. By
610 requiring the same span-to-depth ratio as well as the same number of modular joints to be used for
611 each bridge in the family, the methodology achieves the desired spans using the kit-of-parts and

612 changing only the length of wide flange members. This research also introduced a methodology for
613 achieving rational form variable-depth bridges. By connecting the joints and members at angles,
614 the depth of the structure can vary, allowing more efficient bridge forms to be developed.

615 A sizing optimization approach to minimize the self-weight of structures comprised of the
616 modular joint and wide flange members was proposed. The optimization was implemented for the
617 case study of a 119-m (390-ft) constant-depth simply supported bridge. To ensure transportabil-
618 ity and sufficient structural capacity, geometric and structural constraints were considered. The
619 optimization problem was solved using two stochastic search algorithms: SA and DLS. Results
620 show that DLS and SA are suitable for this optimization problem as both algorithms resulted in
621 nearly equivalent lowest weight solution. The promise of the modular joint is clearly demonstrated
622 through the structural behavior of the lowest weight solution found by the SA algorithm.

623 Ultimately this paper presents a new approach to modular construction in which the joint be-
624 comes the module that is used to achieve a wide range of bridges.

625 This research has focused on the behavior of the system under service and ultimate loads. Fu-
626 ture research should investigate erection strategies and the behavior of the system during erection.
627 Erection strategies such as launching or balanced cantilever are appealing as they do not require
628 as much heavy lifting equipment as conventional truss construction. These techniques cannot typ-
629 ically be used for trusses as conventional gusset plates cannot transmit flexure. The unique char-
630 acteristic of the modular joints being able to carry flexure opens up these alternative and efficient
631 construction strategies. As in the construction of conventional bridges, careful erection analysis
632 must be performed to control the geometry. Fit-up of the connections should also be considered in
633 the erection engineering. The components could be fabricated for either no load fit or steel dead
634 load fit. To promote modularity and a one-size-fits-all approach, no load fit is an appealing strategy
635 as the modular joints could be fabricated identically, regardless of application. Force-fitting would
636 be required in the field, but the splice connections between components could be readily used to
637 achieve the necessary changes to geometry. Camber and geometry control during erection can
638 also be incorporated through the bent flange splice connections between the modular joints and

639 members.

640 **DATA AVAILABILITY STATEMENT**

641 All data, models, or code that support the findings of this study are available from the corre-
642 sponding author upon reasonable request.

643 **ACKNOWLEDGMENTS**

644 This material is based upon work supported by the National Science Foundation under Grant
645 No. CMMI-1351272. Mirela D. Tumbeva is also supported by the O.H. Ammann Research Fel-
646 lowship. Support from these sources and the program managers is gratefully acknowledged.

647 **REFERENCES**

648 AASHTO (2017a). *AASHTO LRFD Bridge Construction Specifications*. American Association of
649 State Highway and Transportation Officials (AASHTO), Washington, D.C., 4th edition.

650 AASHTO (2017b). *AASHTO LRFD Bridge Design Specifications, Customary U.S. Units*. Ameri-
651 can Association of State Highway and Transportation Officials (AASHTO), Washington, D.C.,
652 8th edition.

653 ABAQUS (2016). *ABAQUS/Standard Analysis User's Manual Version 6.14*. Dassault Systemes,
654 Waltham, MA.

655 AISC (2011). *Steel Construction Manual*. American Institute of Steel Construction (AISC),
656 Chicago, IL, 14th edition.

657 Arora, J. S. (2004). *Introduction to Optimum Design*. Elsevier Academic Press, San Diego, CA,
658 2nd edition.

659 Arora, J. S., Huang, M. W., and Hsieh, C. C. (1994). "Methods for optimization of nonlinear
660 problems with discrete variables: a review." *Structural Optimization*, 8(2), 69–85.

661 Bell, E. S. and Medina, R. A. (2019). “Evaluation of gusset-less truss connection to aid bridge
662 inspection and condition assessment.” *Technical Report No. FHWA-NH-RD-26962M*.

663 Chen, J. G., Adams, T. M., Sun, H., Bell, E. S., and Büyüköztürk, O. (2018). “Camera-based
664 vibration measurement of the World War I Memorial Bridge in Portsmouth, New Hampshire.”
665 *Journal of Structural Engineering*, 144(11), 04018207.

666 Covington, E., Engel, C., Kelly-Sneed, K., Noh, J., and Zoli, T. P. (2013). “Portsmouth Memorial
667 Bridge replacement: An exploration of truss design without gusset plates.” *Proceedings of the*
668 *2013 SEI Illinois Chapter Lecture Series*.

669 Department of the Army (1986). “Bailey bridge.” *Field Manual No. 5-277*, Headquarters, Depart-
670 ment of the Army, Washington, DC.

671 Gerbo, E. J., Casias, C. M., Thrall, A. P., and Zoli, T. P. (2016a). “New bridge forms composed of
672 modular bridge panels.” *Journal of Bridge Engineering*, 21(4), 04015084.

673 Gerbo, E. J., Thrall, A. P., Smith, B. J., and Zoli, T. P. (2016b). “Full-field measurement of residual
674 strains in cold bent steel plates.” *Journal of Constructional Steel Research*, 127, 187–203.

675 Gerbo, E. J., Thrall, A. P., and Zoli, T. P. (2020a). “Adjustable bolted steel plate connection:
676 measured behavior of bolts during field installation and numerical parametric investigation.”
677 *Journal of Structural Engineering*, 146(2), 04019189.

678 Gerbo, E. J., Thrall, A. P., and Zoli, T. P. (2020b). “Service and ultimate behavior of adjustable
679 bolted steel plate connections.” *Journal of Structural Engineering*, 146(7), 04020128.

680 Gerbo, E. J., Wang, Y., Tumbeva, M. D., Thrall, A. P., Smith, B. J., and Zoli, T. P. (2018). “Behavior
681 of an adjustable bolted steel plate connection during field installation.” *Journal of Structural*
682 *Engineering*, 144(3), 04017223.

683 Gerbo, E. J., Wang, Y., Tumbeva, M. D., Thrall, A. P., Smith, B. J., and Zoli, T. P. (2019). “Closure
684 to ‘Behavior of an Adjustable Bolted Steel Plate Connection during Field Installation’ by Evan

685 J. Gerbo, Yao Wang, Mirela D. Tumbeva, Ashley P. Thrall, Brian J. Smith, and Theodore P.
686 Zoli.” *Journal of Structural Engineering*, 145(3), 07018015.

687 ISO (2013). *ISO 668: Series 1 freight containers - Classification, dimensions, ratings*. ISO,
688 Switzerland, 6th edition.

689 Joiner, J. H. (2001). *One More River to Cross: The Story of British Military Bridging*. Pen and
690 Sword Books Ltd, South Yorkshire, UK.

691 Keating, P. B. and Christian, L. C. (2012). “Effects of bending and heat on the ductility and fracture
692 toughness of flange plate.” *Technical Report No. FHWA/TX-10/0-4624-2*.

693 Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. (1983). “Optimization by simulated annealing.”
694 *Science, New Series*, 220(4598), 671–680.

695 Mashayekhi, M. and Santini-Bell, E. (2019). “Fatigue assessment of the gusset-less connection
696 using field data and numerical model.” *Bridge Structures*, 15(1-2), 75–86.

697 Medina, J. R. (2001). “Estimation of incident and reflected waves using simulated annealing.”
698 *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 127(4), 213–221.

699 Paya, I., Yepes, V., Gonzalez-Vidoso, F., and Hospitaler, A. (2008). “Mutliobjective optimization
700 of concrete frames by simulated annealing.” *Computer-Aided Civil and Infrastructure Engineer-*
701 *ing*, 23, 596–610.

702 Russell, B. R. and Thrall, A. P. (2013). “Portable and rapidly deployable bridges: Historical
703 perspective and recent technology developments.” *Journal of Bridge Engineering*, 18(10), 1074–
704 1085.

705 Russell, B. R., Thrall, A. P., Padula, J. A., and Fowler, J. E. (2014). “Reconceptualization and
706 optimization of a rapidly deployable floating causeway.” *Journal of Bridge Engineering*, 19(4),
707 04013013.

- 708 Shahsavari, V., Mashayekhi, M., Mehrkash, M., and Santini-Bell, E. (2019). “Diagnostic testing
709 of a vertical lift truss bridge for model verification and decision-making support.” *Frontiers in*
710 *Built Environment*, 5(92), 1–19.
- 711 Thrall, A. P., Adriaenssens, S., Paya-Zaforteza, I., and Zoli, T. P. (2012). “Linkage-based movable
712 bridges: Design methodology and three novel forms.” *Engineering Structures*, 37, 214–223.
- 713 Thrall, A. P., Zhu, M., Guest, J. K., Paya-Zaforteza, I., and Adriaenssens, S. (2014). “Structural
714 optimization of deploying structures composed of linkages.” *Journal of Computing in Civil En-*
715 *gineering*, 28(3), 04014010.
- 716 Wang, Y. T., P., A., and Zoli, T. P. (2016). “Adjustable module for variable depth steel arch bridges.”
717 *Journal of Constructional Steel Research*, 126, 163–173.

718 **List of Tables**

719 1 Definition of angles γ between modular joint and wide flange members. 31

720 2 Optimization results. 32

TABLE 1. Definition of angles γ between modular joint and wide flange members.

Angle notation	Position	Value
γ_i^1	Between M_i and L_{i-1} at point C	Varied within given range
γ_i^2	Between T_{i-1} and U_i at point A'	Varied within given range
γ_i^3	Between N_i and U_i at point B	$\gamma_i^6 - \gamma_i^1 + \gamma_i^5$
γ_i^4	Between T_i and U_i at point A	Varied within given range
γ_i^5	Between M_i and U_i at point B'	$\gamma_i^1 - \alpha_i$
γ_i^6	Between N_i and L_i at point C'	$\arctan \left(\frac{y_{U_i} - (l/2)[\sin \theta + \sin(\theta + \alpha_i)]}{x_{L_i} - x_{U_i} - (l/2)[\cos \theta + \cos(\theta + \alpha_i)]} \right) - \theta$

TABLE 2. Optimization results.

Test	N tests	n (for SA), it (for DLS)	Other Parameters	Results			Lowest weight results								
				μ	σ	C_v	W_{min}	t_1	t_2	t_3	t_4	s_1	s_2	s_3	Run time
				(kN)	(kN)	%	(kN)	(mm)	(mm)	(mm)	(mm)	W14x	Wx14	Wx14	(min)
SA-1	10	1	$v=1, pm=5,$ $m=50, rt=0.8$	1240	43.1	3.48	1201	50.8	12.7	34.9	44.5	233	193	109	9873
SA-2	10	2		1225	18.4	1.51	1201	50.8	12.7	34.9	44.5	233	193	109	13019
DLS-1	10	100	$v=1, pm=5$	1226	12.1	0.99	1208	50.8	12.7	31.8	47.6	233	193	109	6623
DLS-2	10	200		1233	10.2	0.83	1218	41.3	12.7	34.9	44.5	257	193	109	8446

721 **List of Figures**

722 1 Example of the state-of-the-art in modular bridges. Reprinted from Department of
723 the Army (1986). 35

724 2 Memorial Bridge connecting Portsmouth, NH and Kittery, ME: (A) Photograph of
725 the knuckle and (B) Photograph from the deck. Image courtesy of HNTB Corpo-
726 ration. 36

727 3 Fabrication of Memorial Bridge: (A) Cold bending of flange plates and (B) Weld-
728 ing of flange plate to web plate. Image courtesy of HNTB Corporation. 37

729 4 Modular joint. 38

730 5 Modular approach to: (A) Short-span constant-depth bridge, (B) Long-span constant-
731 depth bridge, and (C) Long-span variable-depth bridge. 39

732 6 Connection between modular joints and members: (A) Via straight splice plates,
733 (B) Via adjustable bolted steel plate connection, and (C) Field installation of the
734 adjustable steel plate connection by bolt tightening: untightened (left) and final
735 tightened (right) state [(C) adapted from Gerbo et al. (2019, 2020a), ©ASCE]. . . . 40

736 7 Geometric development of the modular joint: (A) Elevation view of back-to-back
737 modular joints and (B) Section view of one modular joint. 41

738 8 Elevation view of modular joints and wide flange section stacked vertically within
739 the foot print of an ISO container. 42

740 9 Isometric view of family of constant-depth simply supported bridges: (A) 119-m
741 span ($b=3$), (B) 79.2-m span ($b=2$), and (C) 39.6-m span ($b = 1$). 43

742 10 Geometric development of variable-depth bridges: (A) Simply supported bridge
743 and (B) Three-span continuous bridge, angles γ given in Table 1. 44

744 11 Form-finding results for variable-depth bridges: (A) Simply supported bridge and
745 (B) Three-span continuous bridge. Black marks denotes the lowest metric FL^2 for
746 each γ_{max} 45

747 12 Parametric finite element model. 46

748	13	Convergence curves for the lowest weight solution: (A) SA algorithm, and (B)	
749		DLS algorithm.	47
750	14	Response of the structure with the lowest weight solution: (A) Load-displacement	
751		curve from linear inelastic analysis (DL = dead load, LL = live load), (B) Failure	
752		mechanism described by von Mises stress contour at midspan, and (C) Buckling	
753		mode shape corresponding to the critical load factor (smallest eigenvalue) of 3.9. .	48

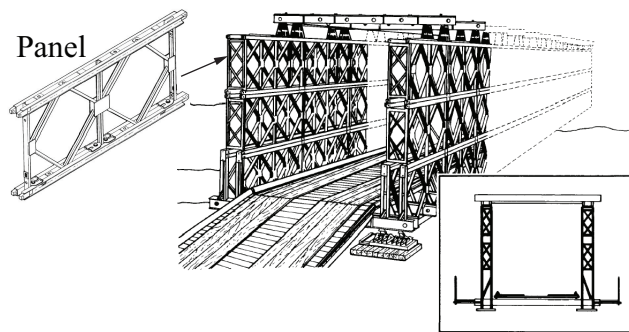
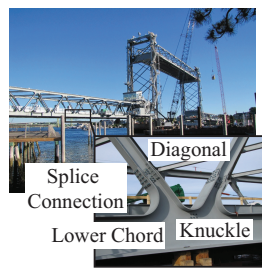
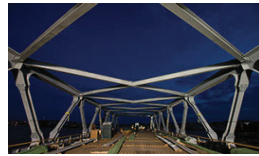


FIG. 1. Example of the state-of-the-art in modular bridges. Reprinted from Department of the Army (1986).



(A)



(B)

FIG. 2. Memorial Bridge connecting Portsmouth, NH and Kittery, ME: (A) Photograph of the knuckle and (B) Photograph from the deck. Image courtesy of HNTB Corporation.



(A)



(B)

FIG. 3. Fabrication of Memorial Bridge: (A) Cold bending of flange plates and (B) Welding of flange plate to web plate. Image courtesy of HNTB Corporation.

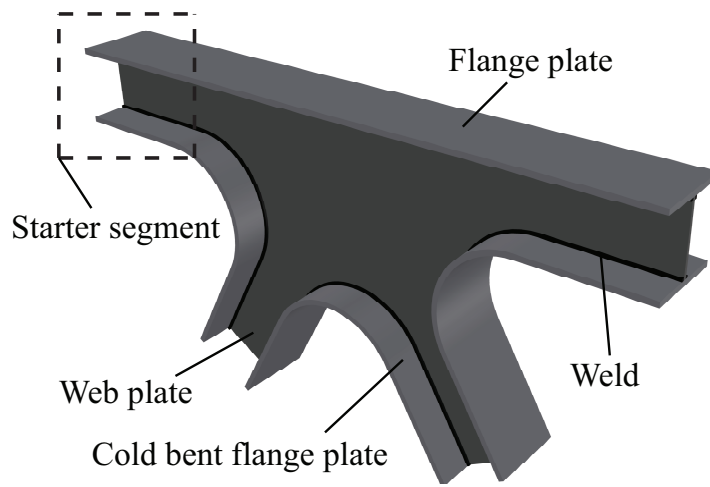


FIG. 4. Modular joint.

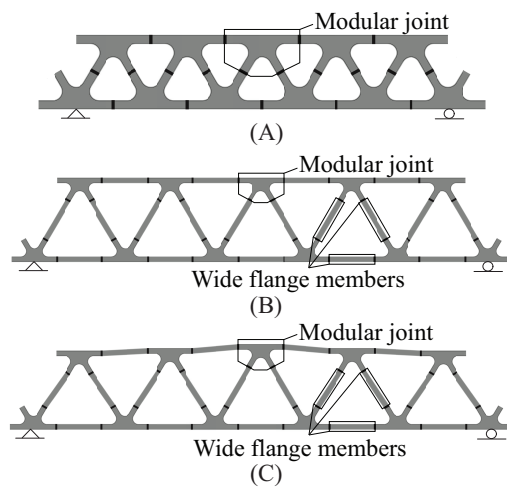


FIG. 5. Modular approach to: (A) Short-span constant-depth bridge, (B) Long-span constant-depth bridge, and (C) Long-span variable-depth bridge.

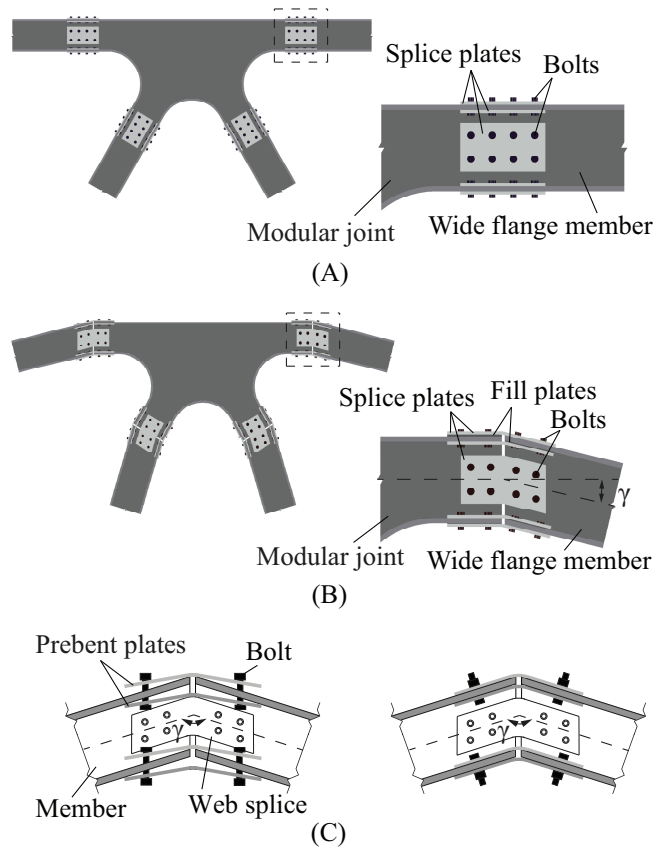


FIG. 6. Connection between modular joints and members: (A) Via straight splice plates, (B) Via adjustable bolted steel plate connection, and (C) Field installation of the adjustable steel plate connection by bolt tightening: untightened (left) and final tightened (right) state [(C) adapted from Gerbo et al. (2019, 2020a), ©ASCE].

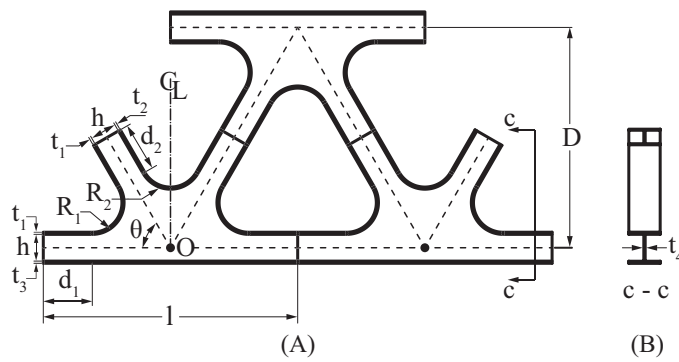


FIG. 7. Geometric development of the modular joint: (A) Elevation view of back-to-back modular joints and (B) Section view of one modular joint.

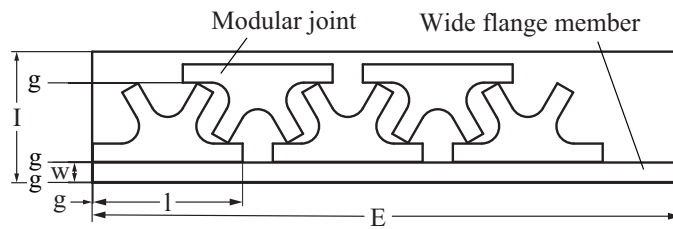


FIG. 8. Elevation view of modular joints and wide flange section stacked vertically within the foot print of an ISO container.

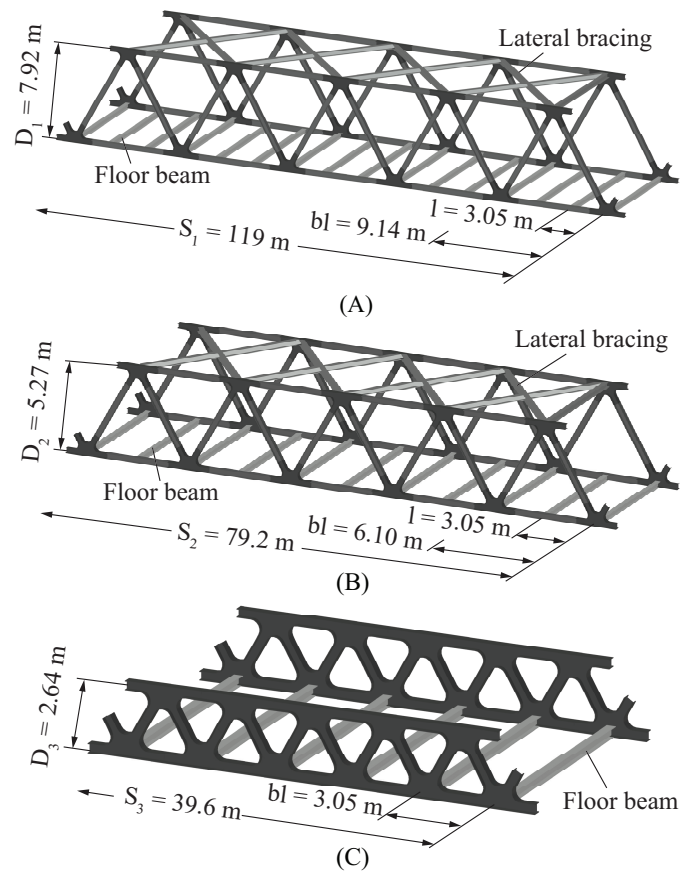


FIG. 9. Isometric view of family of constant-depth simply supported bridges: (A) 119-m span ($b=3$), (B) 79.2-m span ($b=2$), and (C) 39.6-m span ($b = 1$).

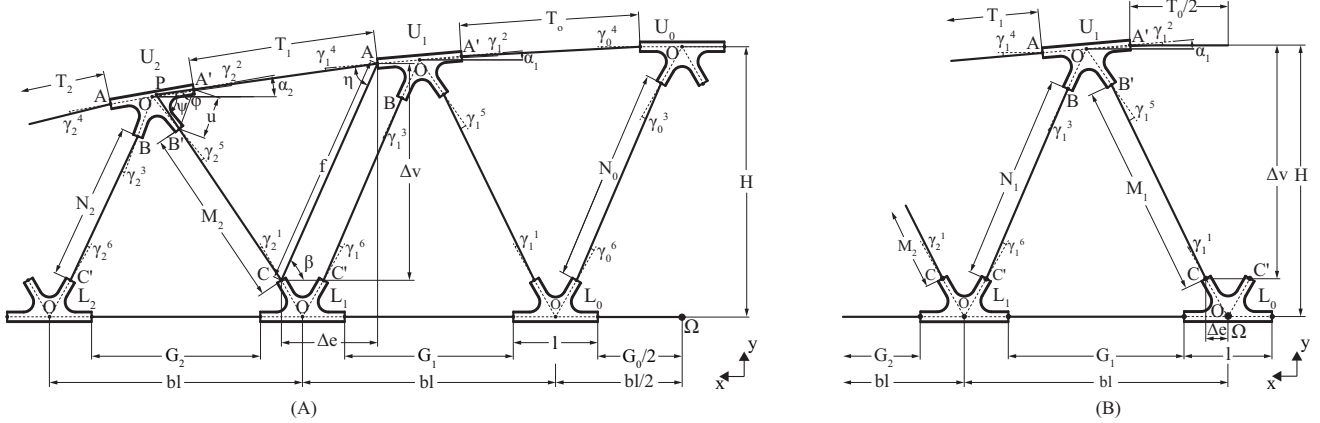


FIG. 10. Geometric development of variable-depth bridges: (A) Simply supported bridge and (B) Three-span continuous bridge, angles γ given in Table 1.

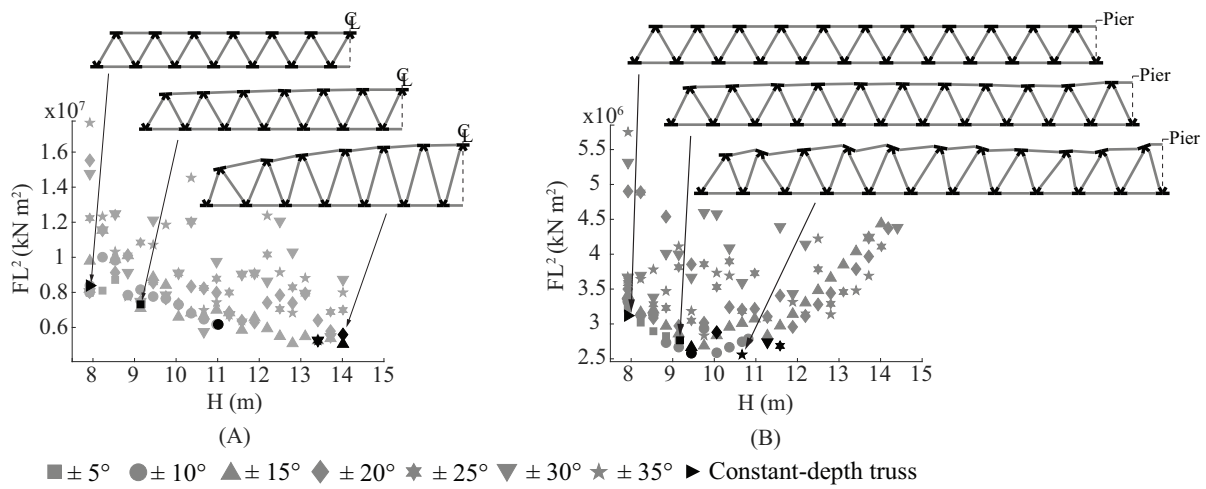


FIG. 11. Form-finding results for variable-depth bridges: (A) Simply supported bridge and (B) Three-span continuous bridge. Black marks denotes the lowest metric FL^2 for each γ_{max} .

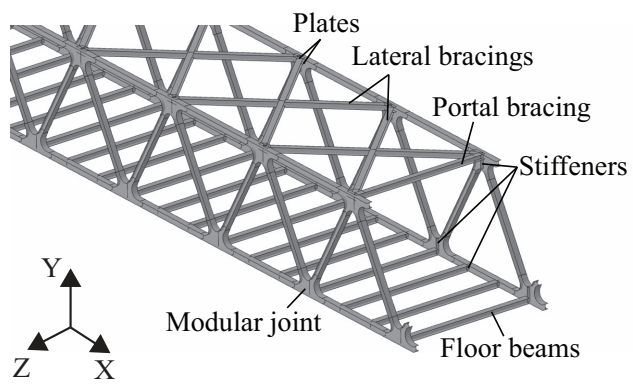


FIG. 12. Parametric finite element model.

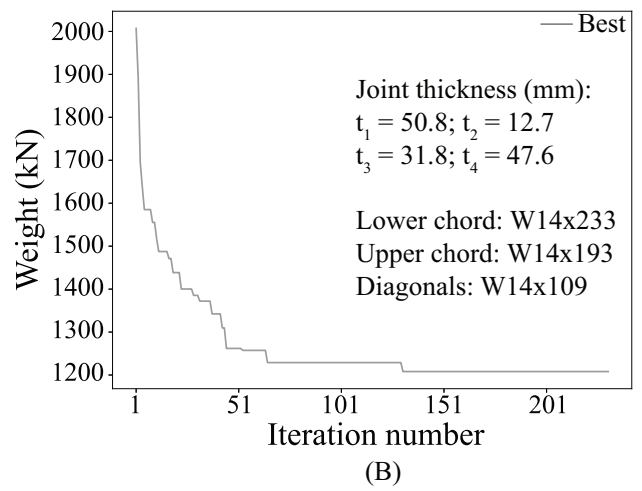
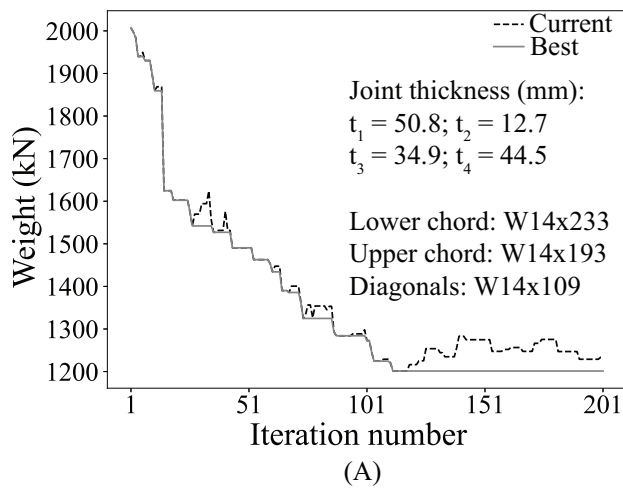


FIG. 13. Convergence curves for the lowest weight solution: (A) SA algorithm, and (B) DLS algorithm.

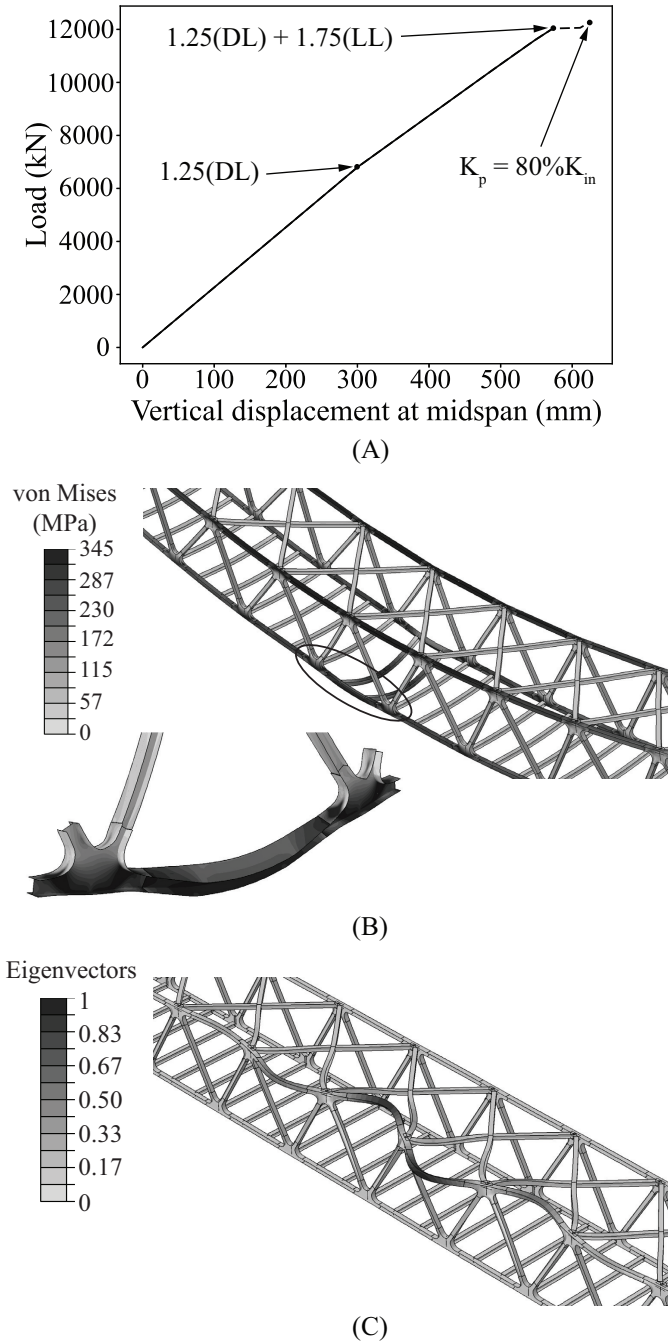


FIG. 14. Response of the structure with the lowest weight solution: (A) Load-displacement curve from linear inelastic analysis (DL = dead load, LL = live load), (B) Failure mechanism described by von Mises stress contour at midspan, and (C) Buckling mode shape corresponding to the critical load factor (smallest eigenvalue) of 3.9.